Gaussian elimination

1. $P(-1;4)$; $Q(1;6)$; $R(3;0)$; $y = ax^2 + bx + c$

$$\left.\begin{array}{l} 4 = a(-1)^2 + b(-1) + c \\ 6 = a\,1^2 + b\,1^2 + c \\ 0 = a\,3^2 + b\cdot 3 + c \end{array}\right| \Rightarrow \left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 1 & 1 & 1 & 6 \\ 9 & 3 & 1 & 0 \end{array}\right) \rightarrow$$

$$\xrightarrow{R_2 = R_2 - R_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 2 & 0 & 2 \\ 9 & 3 & 1 & 0 \end{array}\right) \xrightarrow{R_3 = R_3 - 9R_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 2 & 0 & 2 \\ 0 & 12 & -8 & -36 \end{array}\right) \rightarrow$$

$$\xrightarrow[R_2 = \frac{1}{2}\cdot R_2]{R_3 = R_3 \cdot \frac{1}{4}} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 3 & -2 & -9 \end{array}\right) \xrightarrow{R_3 = 3R_2 - R_3} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 12 \end{array}\right)$$

$$\left.\begin{array}{l} a - b + c = 4 \\ \boxed{b = 1} \\ 2c = 12 \end{array}\right| \begin{array}{l} \boxed{a = b - c + 4 = 1 - 6 + 4 = -1} \\ \\ \Rightarrow \boxed{c = 6} \end{array}$$

$$y = ax^2 + bx + c \Rightarrow y = -x^2 + x + 6$$

Gaussian elimination

(2a.) $-a+2b+c=6$
$a+b+c=-2$
$2a-4b-2c=-6$

$$\left(\begin{array}{ccc|c} -1 & 2 & +1 & 6 \\ 1 & 1 & 1 & -2 \\ 2 & -4 & -2 & -6 \end{array}\right) \longrightarrow$$

$$\xrightarrow{R_2=R_1+R_2} \left(\begin{array}{ccc|c} -1 & 2 & +1 & 6 \\ 0 & 3 & 2 & 4 \\ 2 & -4 & -2 & -6 \end{array}\right) \xrightarrow{R_3=2R_1+R_3} \left(\begin{array}{ccc|c} -1 & 2 & +1 & 6 \\ 0 & 3 & 2 & 4 \\ 0 & 0 & 0 & 6 \end{array}\right)$$

Last row means $0 \cdot a + 0 \cdot b + 0 \cdot c = 6$
$0 \neq 6 \Rightarrow$ no solution

(2b.)
$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -1 & -2 & 0 \\ 2 & 3 & -4 & 0 \end{array}\right) \xrightarrow{R_2=R_1-R_2} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 2 & 0 & 0 \\ 2 & 3 & -4 & 0 \end{array}\right) \xrightarrow{R_3=R_3-2R_1}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \xrightarrow{R_3=R_2-2R_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

more then one solution

$x+y-2z=0$
$2y+0 \cdot z=0$
$0 \cdot z=0$ $\Rightarrow$ $z=\lambda, \lambda \in \mathbb{R}$

$y=0$
$x=2\lambda$

(2c.) $x-2y+3z-2t=15$
$2x+3y-z-4t=2$
$6x+16y-10z-12t=-22$

$$\left(\begin{array}{cccc|c} 1 & -2 & 3 & -2 & 15 \\ 2 & 3 & -1 & -4 & 2 \\ 6 & 16 & -10 & -12 & -22 \end{array}\right)$$

$R_3 = R_3 \cdot \frac{1}{2}$

$$\left(\begin{array}{cccc|c} 1 & -2 & 3 & -2 & 15 \\ 2 & 3 & -1 & -4 & 2 \\ 3 & 8 & -5 & -6 & -11 \end{array}\right) \xrightarrow{R_2 = R_2 - 2R_1} \left(\begin{array}{cccc|c} 1 & -2 & 3 & -2 & 15 \\ 0 & 7 & -7 & 0 & -28 \\ 3 & 8 & -5 & -6 & -11 \end{array}\right)$$

$$\xrightarrow[R_2 = \frac{R_2}{7}]{R_3 = R_3 - 3R_1} \left(\begin{array}{cccc|c} 1 & -2 & 3 & -2 & 15 \\ 0 & 1 & -1 & 0 & -4 \\ 0 & 14 & -14 & 0 & -56 \end{array}\right) \xrightarrow{R_3 = \frac{R_3}{14}} \left(\begin{array}{cccc|c} 1 & -2 & 3 & -2 & 15 \\ 0 & 1 & -1 & 0 & -4 \\ 0 & 1 & -1 & 0 & -4 \end{array}\right)$$

$$\xrightarrow{R_3 = R_3 - R_2} \left(\begin{array}{cccc|c} 1 & -2 & 3 & -2 & 15 \\ 0 & 1 & -1 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{array}{l} x - 2y + 3z - 2t = 15 \\ y - z = -4 \\ 0 \cdot t = 0 \end{array}$$

$\Downarrow$

more then one solution

parameter λ, μ ; $\boxed{t = \lambda}$; $\boxed{z = \mu}$; $\lambda, \mu \in \mathbb{R}$

$\Rightarrow \boxed{y = \mu - 4}$

$x = 2y - 3z + 2t + 15 = 2\mu - 8 - 3\mu + 2\lambda + 15 =$

$\boxed{x = 7 - \mu + 2\lambda}$

3.

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 1 & -1 & -2 & 2 \\ 2 & 3 & z & t \end{array}\right) \xrightarrow{R_2 = R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 2 & 0 & -1 \\ 2 & 3 & z & t \end{array}\right) \xrightarrow{R_3 = 2R_1 - R_3}$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 2 & 0 & -1 \\ 0 & -1 & (-4-z) & (2-t) \end{array}\right) \xrightarrow{R_3 = R_2 + 2R_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & -(8+2z) & (-1+4-2t) \end{array}\right)$$

a) $-8 - 2z \neq 0 \quad z \neq -4$

b) $-8 - 2z = 0$; $z = -4$ and $3 = 2t$; $t = \frac{3}{2}$

c) $-8 - 2z = 0$; $z = -4$ and $t \neq \frac{3}{2}$