Exercise 1: [A12] By calculating the parameters m_1, n_1 and m_2, n_2, show that the vectors

$$\vec{a} = \begin{pmatrix} 1 \\ 7 \\ -9 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

satisfy the equations $\vec{b} = m_1\vec{a} + n_1\vec{c}$ and $\vec{c} = m_2\vec{a} + n_2\vec{b}$.

Exercise 2: [A13] Given are $\vec{a} = 3\vec{e}_x + 2\vec{e}_y - 5\vec{e}_z$ and $\vec{b} = 2\vec{e}_x - 4\vec{e}_y + \vec{e}_z$. Calculate $2\vec{a} + 4\vec{b}$ and $3\vec{a} - 2\vec{b}$.

Exercise 3: [A16] a) Which combinations out of the following vectors are collinear?

$$\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} \quad \vec{d} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

b) Which combinations out of the following vectors are coplanar?

$$\vec{a} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 15 \\ 2 \\ -1 \end{pmatrix} \quad \vec{d} = \begin{pmatrix} -9 \\ -4 \\ 5 \end{pmatrix}$$

Exercise 4: [A17]

a) Determine values for x and z, so that $\vec{a} = \begin{pmatrix} 4 \\ -2 \\ -3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} x \\ 4 \\ z \end{pmatrix}$ are collinear.

b) Determine values of y such that the vectors $\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} 1 \\ y \\ -9 \end{pmatrix}$ are coplanar.

Exercise 5: [A18] Show that force $\vec{F} = \begin{pmatrix} -12\text{ N} \\ 1\text{ N} \\ 10\text{ N} \end{pmatrix}$ can be decomposed in the direction of $\vec{a} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix}$. The vectors $\vec{F_a}$ and $\vec{F_b}$ are the partial forces in direction of $\vec{a}$ and $\vec{b}$ respectively. Calculate them.

Exercise 6: [A19] Given are $\vec{a} = \begin{pmatrix} 1 \\ 8 \\ -4 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ 7 \\ 4 \end{pmatrix}$.

a) Determine the length of the given vectors.

b) Determine the unit vectors.

c) What is the angle between the vectors?

Exercise 7: [A20] a) Show that $\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} -5 \\ 2 \\ 3 \end{pmatrix}$ are perpendicular.

b) For which y are $\vec{a} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 6 \\ y \\ 1 \end{pmatrix}$ perpendicular?

Exercise 8: [A21] Given are $\vec{a} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$

a) Calculate the angle between $\vec{a}$ and $\vec{b}$.

b) Calculate $\vec{a} \cdot \vec{e}_x$ and $\vec{b} \cdot \vec{e}_y$.

c) What is the angle between $\vec{a}$ and the$x$-axis as well as the angle between $\vec{b}$ and the y-axis?

Exercise 9: [A22] For which y, z is the vector $\vec{c} = \begin{pmatrix} 3 \\ y \\ z \end{pmatrix}$ perpendicular to both vectors $\vec{a} = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$?

Exercise 10: [A23b] Vector $\vec{b}_a$ is called projection vector of $\vec{b}$ in the direction of $\vec{a}$. Determine the projection vectors $\vec{b}_a$ and $\vec{a}_b$ for $\vec{a} = \begin{pmatrix} 10 \\ 5 \\ 10 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 8 \\ 4 \end{pmatrix}$.

Exercise 11: [A24] Given the vectors $\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 7 \\ 4 \\ -2 \end{pmatrix}$. Determine a vector that is perpendicular to both $\vec{a}$ and $\vec{b}$. How many such vectors do exist?

Exercise 12: [A26] For $\vec{a} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$ calculate $(4\vec{a} + 3\vec{b}) \times (2\vec{a} - 4\vec{b})$ by

a) calculating the terms in brackets and multiply.

b) applying distributive law.

Exercise 13: [A32] If the box product $\vec{c} \cdot (\vec{a} \times \vec{b}) = \vec{0}$, then $\vec{a}, \vec{b}, \vec{c}$ are coplanar. Examine whether the given vectors are coplanar. If possible find λ, μ such that $\vec{c} = \lambda\vec{a} + \mu\vec{b}$ is satisfied.

a) $\vec{a} = \begin{pmatrix} -2 \\ 7 \\ 1 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$

b) $\vec{a} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$

Exercise 14: [A36]

a) Determine a parametric equation of the straight line through $P_1(2, -4, 3)$ and $P_2(1, -12, -3)$.

b) Examine, which of the points $P_3(4, 12, 15)$ and $P_4(-1, 3, 0)$ belong to the straight line.

Exercise 15: [A37] Given are the straight lines

$$g_1 : \vec{r}_1 = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} + t_1 \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \qquad g_2 : \vec{r}_2 = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} + t_2 \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$$

$$g_3 : \vec{r}_3 = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix} + t_3 \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \qquad g_4 : \vec{r}_4 = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} + t_4 \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$$

Which are parallel, which coincide?

Exercise 16: [A40] Given are the four points $P_1(2,-1,-2), P_2(8,3,-4), P_3(1,-7,3)$ and $P_4(-4,3,-7)$.

a) Show that the four points lie in the same plane.

b) Determine the intersection point of the straight lines P_1P_2 and P_3P_4 as well as the (acute) angle between the straight lines.

Exercise 17: [A41] Given is the straight line $\vec{r} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$.

a) Give reasons why the straight line lies in the x-y-plane.

b) Transform the parametric equation of the straight line into the form $y = m \cdot x + n$.

Exercise 18: [A45] Examine whether the straight lines $\vec{r}_1 = \begin{pmatrix} -7 \\ -1 \\ 4 \end{pmatrix} + \lambda_1 \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ and $\vec{r}_2 = \begin{pmatrix} -2 \\ -3 \\ 14 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix}$ lie in the same plane. Determine a parametric equation of the plane if possible.

Exercise 19: [A47] The straight lines $g_1 : \vec{r}_1 = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix} + \lambda_1 \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$ and $g_2 : \vec{r}_2 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 10 \\ 4 \end{pmatrix}$ are parallel. Determine a parametric equation of the plane that is spanned by the straight lines g_1 and g_2.

Exercise 20: [A48] Determine the intersection point of the plane P and the straight line L.

$$P : \vec{r} = \begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \qquad L : P_1(-10,-10,6)\ ;\ P_2(-3,4,-15)$$

Exercise 21: [A58] Given the points $P_1(7,5,8)$, $P_2(11,20,10)$ and $P_3(1,-16,6)$. Determine an equation of the plane through P_1, P_2 and P_3

a) in parametric form.

b) in normal form.

c) in Hesse normal form.

Exercise 22: [A60] Given the three planes

$$P_1 : \vec{r} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$$

$$P_2 : -9 \cdot x + y + 7 \cdot z = 12$$

$$P_3 : \begin{pmatrix} 18 \\ -2 \\ -14 \end{pmatrix} \vec{r} - 100 = 0$$

What is the position of these planes relative to each other?

Exercise 23: [A61] Given the plane $12 \cdot x - 4 \cdot y + 3 \cdot z = 26$.

a) Calculate its distance from the origin.

b) What is the distance of the point $P_1(36, -13, 19)$ and the plane?

Exercise 24: [A62] Given the point $P_1(3, 4, 4)$. Determine an equation of the plane, which contains the point P_1 and is perpendicular to the y-axis.

Exercise 25: [A63] Given the plane $P :\ 6x - 2y - 3z = 14$. Determine an equation of the plane, which is parallel to P and has a distance $d = 4$ from P.

Exercise 26: [A64] Determine the vertexes and the length of the heights of the triangle given by the three straight lines $l_1 : \vec{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, $l_2 : -2 \cdot x + y = 3$ and $l_3 : \begin{pmatrix} 3 \\ 2 \end{pmatrix} \vec{r} = 22$.